A GRAPHICAL METHOD FOR ESTABLISHING THE ACTIVE CURVE OF A METEOR RADIANT POINT ON A GIVEN PATH

N. N. Kalinina

ABSTRACT. A graphical method for determining the positions of reflecting points of the active curve of a radiant point is described.

A meteor radiant point above the plane of the horizon of a given path point, N*[1], appears on the path such that at each moment in the time considered meteor trails associated with it can reflect a radio signal at points forming a curve on a sphere that K. V. Kostylev has proposed we call the active radiant curve [1, 2] (the radius is R + h, where R is the radius of the earth, and h is the mean altitude of the meteor zone). /89

In order to solve certain of the basic problems of meteor burst propagation, and particularly those concerned with forecasting the number of meteor reflections and the communication line duty cycle, we must begin with a determination of geometric parameters of that propagation directly associated with the reflecting points of the active radiant curve.

A strictly analytical calculation of the coordinates of reflecting points for the particular radiant point reduces to an eighth-degree trigonometric equation, precise solution of which is possible only by using a computer, something that is not always possible. An approximate solution, using conventional computational means, takes a great deal of time and effort.

The proposed graphical method of determining the positions of the reflecting points of the active radiant curve on the R + h sphere, and, as a result, their coordinates, is relatively simple, takes little time for construction, and is reasonably accurate. It is based on satisfying the following three basic conditions for meteor burst propagation:

1. the reflecting point in the meteor trail should be visible from both end points of the path, T and R;

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1. Path point N is the foot of a perpendicular dropped from the center of the earth to the path line.

2. this point should be at altitude h from the terrestrial surface (in the meteor zone), that is, belong to a sphere of radius R + h; /90

3. it should be the point of tangency of the meteor trail and the specular ellipsoid (the ellipsoid providing the specular reflection of the radio waves), the foci of which coincide with points T and R.

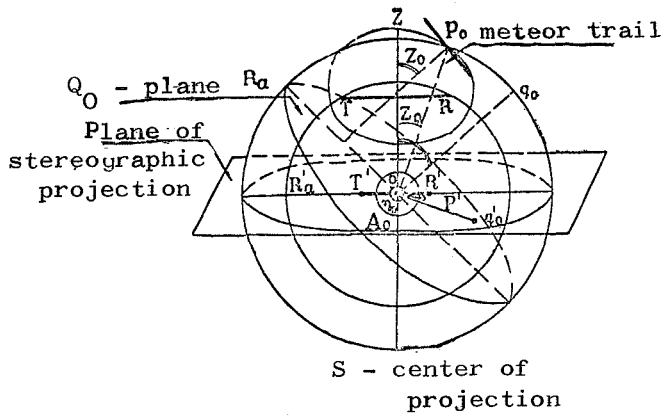


Figure 1. Geometry of the location of reflecting points of a meteor trail and of the Q-plane pole on a sphere in a meteor zone and on the plane of a stereographic projection.

Let us call the auxiliary plane passing through an arbitrary radiant point, R_a, and the center of the earth, O, the Q-plane (Figure 1). It is obvious that as many such planes as desired (a sheaf of planes) can be drawn through the "radiant-center of the earth" line.

The intersection of the normal to the Q-plane, drawn from point O, and the R + h sphere yields a point that is pole q of plane Q.

Let us select that one of the planes from the sheaf of Q-planes, the pole of which and the specified radiant point, R_a, lie on the same great circle passing through the path's zenith point, N. Let us call it the Q_o-plane, and its pole q_o. The spherical coordinates of the pole will be Z_oA_o for constant radius of the R + h sphere. No further reference to it will be made.

Let us take one of the set of reflecting points forming the active curve for the specified radiant point belonging to the meteor trail in the plane tangent to the specular ellipsoid and parallel to the Q_o-plane, /91

designated in Figure 1 as p_0. Now let us take $\zeta_0\eta_0$ for the spherical coordinates of point p_0.

Pole q_0 in Figure 1 does not coincide with reflecting point p_0. Angle Z_0 is larger than angle ζ_0, except when $Z_0 = \zeta_0 = 0$. This can be explained by the fact that ζ_0 is the angle between the bearing to the zenith point N of the path and the radius of the R + h sphere (or to the normal to the sphere) at point p_0, and Z_0 is the angle between this same bearing to the zenith and the normal to the Q_0-plane at point O, or, what is the same thing, the normal to the Q_0-plane passing through the point p_0 and which, in turn, is the normal to the specular ellipsoid, the curvature of which is greater than the curvature of the R + h sphere. Point p_0 can lie only in a region suitable for meteor burst propagation bounded by the planes of the horizons of points T and R, and ζ_0 can change from 0 to 10°05'09" (when the radius of the earth is R = 6371.21 km [3]), whereas Z_0 can take values from 0 to 90°.

Nor can the azimuths η_0 and A_0, read from the path line on the R point side, coincide at points p_0 and q_0. But there is a definite analytical connection, obtained in the following form, between coordinates $\zeta_0\eta_0$ and Z_0A_0 of these points

$$\cos Z_0 = \frac{\bar{a}^2\left(\sec\frac{\gamma_m}{2}\cos\zeta_0 - \right.}{\sqrt{\left\{\bar{b}^2\left[\sec\frac{\gamma_m}{2}\sin\zeta_0\cos\eta_0 + \frac{1}{2}(\sec\sigma_T\sin\gamma_T - \sec\sigma_R\sin\gamma_R)\right]\right\}^2 + \left.-\sec\sigma_T\cos\gamma_T\right)} \qquad (1)$$
$$+\bar{a}^4\sec^2\frac{\gamma_m}{2}\sin^2\zeta_0\sin^2\eta_0 + \left[\bar{a}^2\left(\sec\frac{\gamma_m}{2}\cos\zeta_0 - \sec\sigma_T\cos\gamma_T\right)\right]^2},$$

$$\cos A_0 = \frac{\bar{b}^2\left[\sec\frac{\gamma_m}{2}\sin\zeta_0\cos\eta_0 + \frac{1}{2}(\sec\sigma_T\sin\gamma_T - \right.}{\sqrt{\left\{\bar{b}^2\left[\sec\frac{\gamma_m}{2}\sin\zeta_0\cos\eta_0 + \frac{1}{2}(\sec\sigma_T\sin\gamma_T - \sec\sigma_R\sin\gamma_R)\right]\right\}^2 + \left.-\sec\sigma_R\sin\gamma_R)\right]} \qquad (2)$$
$$+\bar{a}^4\sec^2\frac{\gamma_m}{2}\sin^2\zeta_0\sin^2\eta_0},$$

where /92

a and b are the semimajor and semiminor axes of the specular ellipsoid of rotation divided by the radius of the earth;

$\gamma_m/2 = 10°05'09''$ when the radius of the earth is R = 6371.21 km;

γ_T and γ_R are segments of the path from points T and R to point N, in measurement degrees;

σ_T and σ_R are the visual angles from the center of the earth to points T and R on the path and, correspondingly, the points of tangency of the horizon lines for these points, T and R, with the terrestrial surface.

There is, thanks to this relation, a unique correspondence between $\zeta_0\eta_0$ and Z_0A_0. Using it, it is possible to match a Z_0A_0 point with each $\zeta_0\eta_0$ point, and we can construct a stereographic grid with Z_0 = constant, A_0 = constant based on the $\zeta_0\eta_0$ stereographic grid and using the method described in [3]. This we shall call the Z_0A_0 coordinate grid.

The Z_0A_0 coordinate grid calculated for one path is not suitable for another because its appearance will depend on the path length [see Eqs. (1) and (2)]. The Z_0A_0 coordinate grid within the limits of a permissible region will have the appearance of that in Figure 2 for a plain path 700 km in length.

Let us now assign the coordinates $Z_1 = Z$ and $A_1 = A_0^* - A$ to the radiant point. Here A_0^* is the azimuth of the bearing to the R point on the path, and Z and A are the horizontal celestial coordinates of the radiant point.

Plane Q_0 in Figure 3 is perpendicular to the plane of the drawing and therefore is shown as the straight line R_aO. We see that radiant point R_a and pole q_0 are disposed at azimuths A_1 and that

$$A_0 = A_1 + 180°, \tag{3}$$

while the sum of their zenith distances is

$$Z_1 + Z_0 = 90°. \tag{4}$$

The Z_0A_0 coordinate grid is a grid of the coordinates of poles

q_0 of planes Q_0 of all possible radiant points, so this grid can be used to plot on tracing paper the pole q_0 in Z_0 and A_0 coordinates, found using Eqs. (3) and (4). Thus, the Z_0A_0 coordinate grid, because of the unique correspondence of points p_0 and q_0, is used to find the one reflecting point corresponding to the given radiant point.

We must, in order to use the Z_0A_0 grid when determining the other reflecting points of the active curve for the specified radiant point, find the coordinates of the poles of the other Q-planes of the sheaf belonging to the specified radiant point. /94

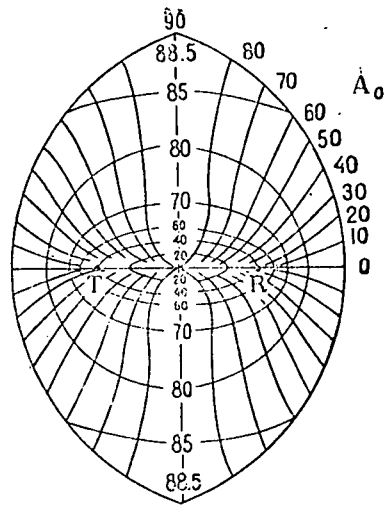


Figure 2. Z_0A_0 coordinate grid for a 700 km long plain path.

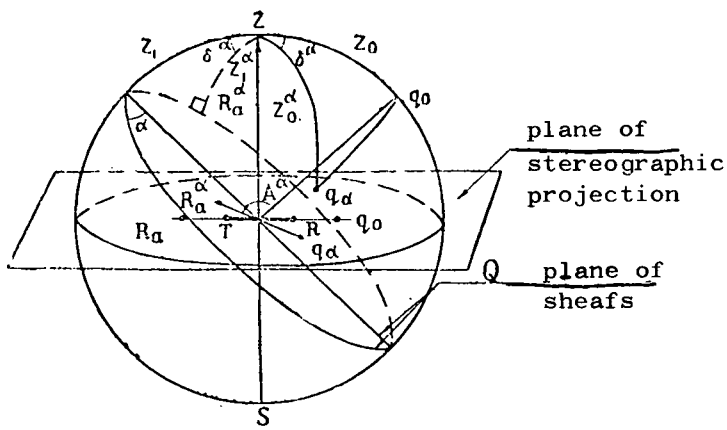


Figure 3. Geometry of the position of pole q_α of plane Q_α of a sheaf positioned at angle α to the initial Q-plane (on the sphere and plane of a stereographic projection).

Let the Q_α-plane of the sheaf be found at angle α to the Q_0-plane. It is obvious that the position of point R_a^α always can be found such that an arc struck through R_a^α and the zenith point N of the path, once again will pass through pole q_α, which will be at a 90° angle to R_a^α, such that

$$Z_1 + Z_0^\alpha = 90°; \tag{5}$$

$$LR_aZR_a^\alpha = Lq_0Zq_\alpha = \delta_\alpha; \tag{6}$$

$$A_0^\alpha = 180 + (A_1 \pm \delta^\alpha), \tag{7}$$

if $h_T = h_R$, or $h_T \neq h_R$ when $Z_1 < 90 - Z_0^0$ and

$$A_0^\alpha = A_1 \pm \delta^\alpha, \tag{7a}$$

if $h_T \neq h_R$ when $Z_1 \geq 90 - Z_0^0$, where $Z_0^0 = \left|\frac{\gamma_R - \gamma_T}{2}\right|$.

Angles t_0^α and δ^α can be found from

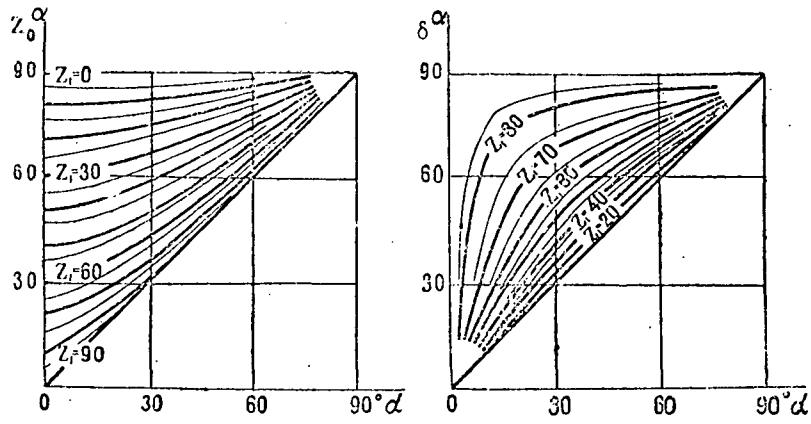
$$\cos Z_0^\alpha = \sin Z_1 \cdot \cos\alpha; \tag{8}$$

$$\cos\delta_\alpha = \operatorname{ctg} Z_1 \cdot \operatorname{ctg} Z_0^\alpha, \tag{9}$$

obtained from the spherical triangle $R_aZR_a^\alpha$ (see Figure 3). Thus, we obtain the values of the coordinates of the poles of the Q_α-plane corresponding to the specified radiant point for a series of α values.

Z_0^α and δ^α were calculated for different Z_1 using Eqs. (8) and (9) as a matter of convenience and then were used to plot curves (Figures 4 and 5). The curves plotted for $Z_1 = 0 - 90°$ in 5° increments are universal curves suitable for any path. One of the Z_0^α coordinates of pole q^α, corresponding to the specified radiant point, and the angle δ^α between the arcs on which poles q_0 and q_α are located, can be found directly from the curves for specified Z_1. /95

Angle δ^α was used to plot the poles, or a second coordinate of the pole, A_0^α, was found using it and Eq. (7).



Figures 4 and 5. Zenith distance of the radiant point, Z_1, zenith distance of the pole of the Q_α-plane, Z_0^α, and angle δ^α in terms of the angle of inclination of plane Q_α of sheaf α.

Then, as has already been mentioned, poles q_α were plotted on tracing paper according to the values of Z_0^α, δ^α, or Z^α, and $A^\alpha{}_0$, using the Z_0A_0 coordinate grid. Connecting them we obtain a curve which, because of the unique correspondence between poles and reflecting points, and the construction of the Z_0A_0 coordinate grid corresponding to it, will be the active curve for the specified radiant point. If necessary, the coordinates $\zeta_0\eta_0$ of the reflecting points of this curve can be determined graphically from the $\zeta_0\mu_0$ coordinate grid, from which the Z_0A_0 coordinate grid had been constructed earlier. The two grids are constructed separately in order not to clutter up the drawing.

The method was checked by constructing five active curves with h= 100 km /96 for radiant points with coordinates Z_1 and A_1, equal to 20° and 0°, 20° and 90°, 85° and 10°, 85° and 30°, 50° and 60° (Figure 6, the reflecting points of the active curves are marked x) for a plain path 700 km long, the Z_0A_0 coordinate grid for which is shown in Figure 2. It is 40 cm in diameter (accuracy is adequate).

The five active curves were compared with the active curves for these same radiant points and for the same path, calculated using different formulas and a computer (Figure 6, the reflecting points of the curves are shown by dots). K. V. Kostylev provided us with the data for these active curves.

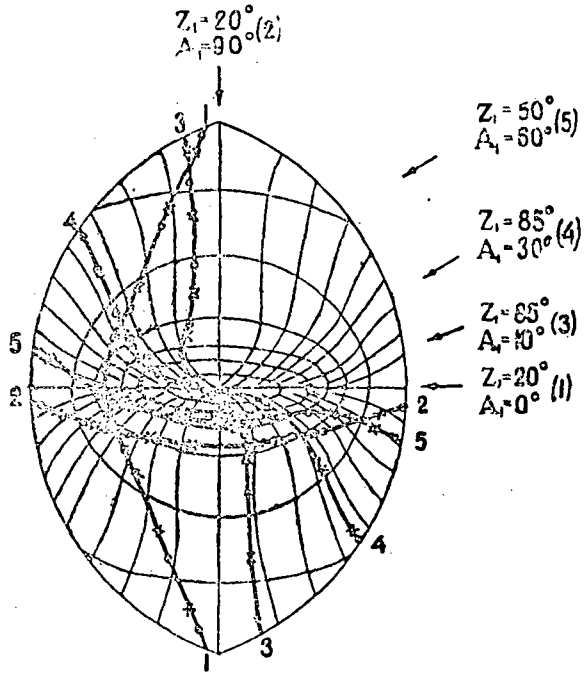


Figure 6. A comparative plot of active curves for five different radiant points, calculated on a computer (.) and obtained by the method cited (x).

Comparison of the corresponding curves in Figure 6 shows no significant deviations.

The accuracy with which the active radiant point curves are plotted using the proposed method will depend on the scope and accuracy of the calculation, and on the care with which the Z_0A_0 coordinate grid and Figures 4 and 5 are constructed. /97

The proposed method can be used to plot a curve from 15 to 30 points in 5 to 10 minutes, and this is comparable in time to that required to calculate them using a "Nairi" computer. It takes about 7 to 10 hours to calculate the number of points indicated for an active curve using conventional calculating machines.

The following conclusions can be drawn from the foregoing.

1. A new graphical method of determining the active radiant curve is proposed, one making it possible to quite rapidly and simply find the positions of reflecting points on an R + h sphere.

2. The graphical method of determining the active radiant curve, together with the graphical method for determining the geometric parameters of meteor burst propagation [3], enables us to solve a number of problems that earlier required extremely cumbersome calculations.

In conclusion, the author expresses appreciation to Lecturer K. V. Kostylev for providing the material needed to check the method, for reviewing the manuscript, and for valuable advice.

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